MeshDMP: Motion Planning on Discrete Manifolds using Dynamic Movement Primitives

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With industry 4.0, manufacturing transitioned from mass production to **mass customization**, thus requiring robotic systems to constantly **adapt to new requirements**, and traditional programming techniques might not be sufficient to cope with **contact-rich tasks** on **complex surfaces**. With this work we propose **MeshDMP**, a DMP-based framework which enables the learning of motion policies directly on surfaces described as polygonal meshes.

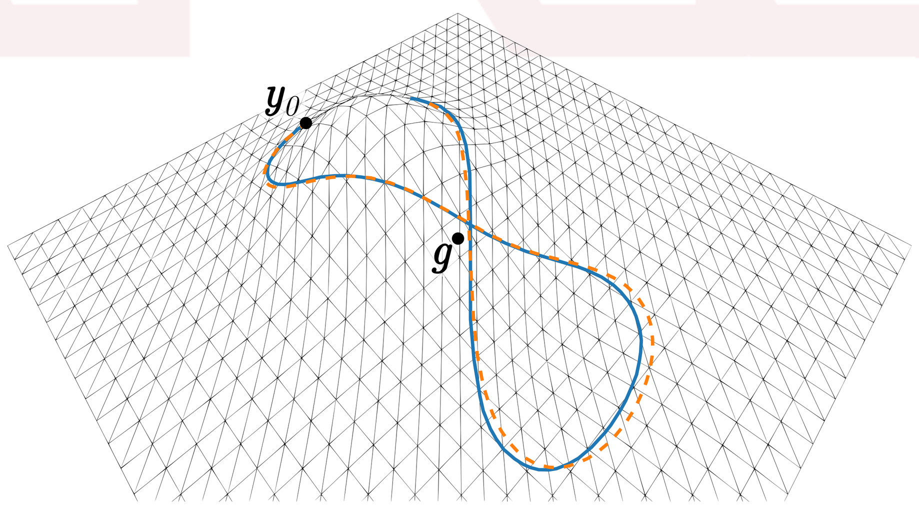
Methodology

Rhythmic DMPs on Manifolds

Rhythmic DMPs are non-linear oscillators that can be learned from demonstration; when the **state** $\boldsymbol{y}$ lies on a **Riemann manifold** M [1]

$$\begin{aligned} \nabla_{\boldsymbol z} \boldsymbol z &= \Omega\left(\alpha\left(\beta \operatorname{Log}_{\boldsymbol y}(\boldsymbol g) - \boldsymbol z\right) + \boldsymbol f(\phi)\right),\\ \dot{\boldsymbol y} &= \Omega \boldsymbol z,\\ \dot\phi &= \Omega.\end{aligned} \tag{1}$$

This formulation allows to **encode any periodic motion**. Our goal is to achieve the following: learn and reproduce motion on surfaces.



Differentiable and Discrete Manifolds

- ▷ Riemann manifolds are smooth;
- ▷ smooth surfaces $\mathcal{S}$ are generally not tractable;
- ▷ we want to use an approximate representation: a mesh $\mathcal{M}$.

Geodesics

- ▷ on $\mathcal{S}$, a curve $\gamma_{\boldsymbol x_1\to \boldsymbol x_2}$ is geodesic if it minimises the distance on the surface;
- ▷ on $\mathcal{M}$, we can use numerical algorithms [2] to compute shortest path curves between $\boldsymbol m_1, \boldsymbol m_2 \in \mathcal{M}$:

$$\Gamma_{\boldsymbol m_1\to \boldsymbol m_2} = \{\overline{\boldsymbol p_1\boldsymbol p_2}, \overline{\boldsymbol p_2\boldsymbol p_3}, \dots, \overline{\boldsymbol p_{n-1}\boldsymbol p_n}\} \tag{2}$$

Logarithmic Map

- ▷ on $\mathcal{S}$, $\boldsymbol v = \operatorname{Log}_{\boldsymbol x_1} : \mathcal{S} \to T_{\boldsymbol x_1}\mathcal{S}$ projects a point $\boldsymbol x_2$ into $T_{\boldsymbol x_1}\mathcal{S}$ such that $|\boldsymbol v| = |\gamma_{\boldsymbol x_1\to \boldsymbol x_2}|$ and $\boldsymbol v$ has direction $\dot\gamma_{\boldsymbol x_1\to \boldsymbol x_2}(\boldsymbol x_1)$;
- ▷ on $\mathcal{M}$, borrowing the above definition, and given the geodesic Γ on $\mathcal{M}$ (2):

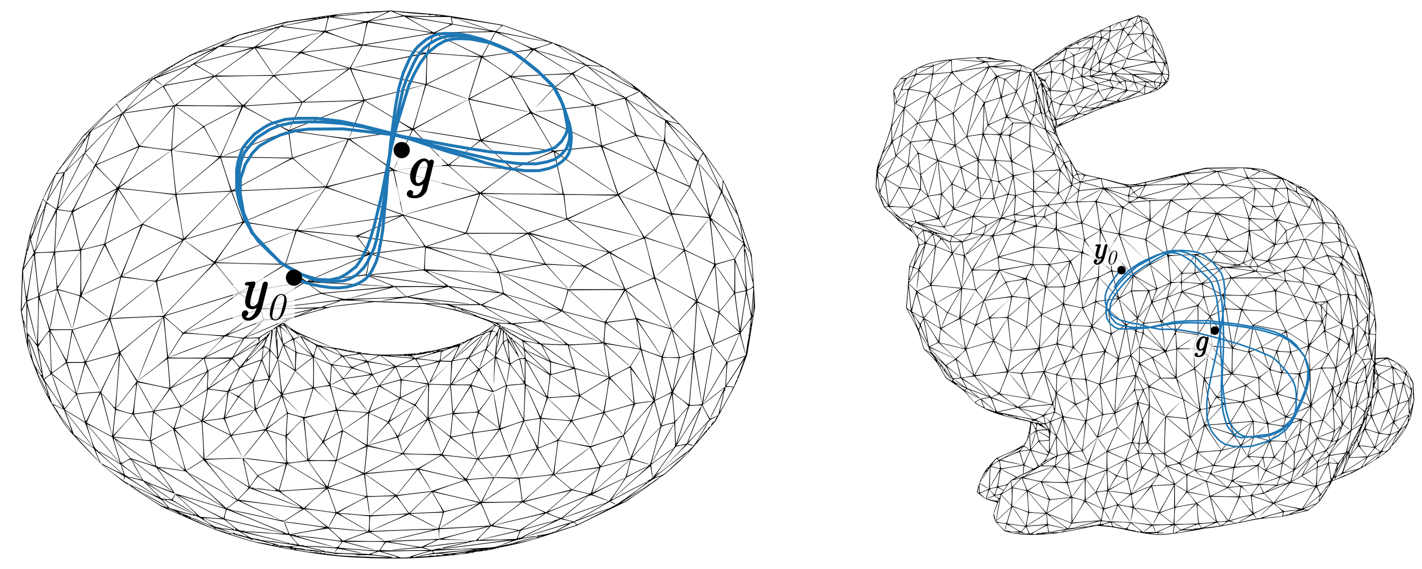
$$\operatorname{Log}_{\boldsymbol m_1}(\boldsymbol m_2) := \frac{\overline{\boldsymbol p_1\boldsymbol p_2}}{\|\overline{\boldsymbol p_1\boldsymbol p_2}\|}\sum_{i=1}^{n-1}\left\|\overline{\boldsymbol p_i\boldsymbol p_{i+1}}\right\|. \tag{3}$$

Exponential Map

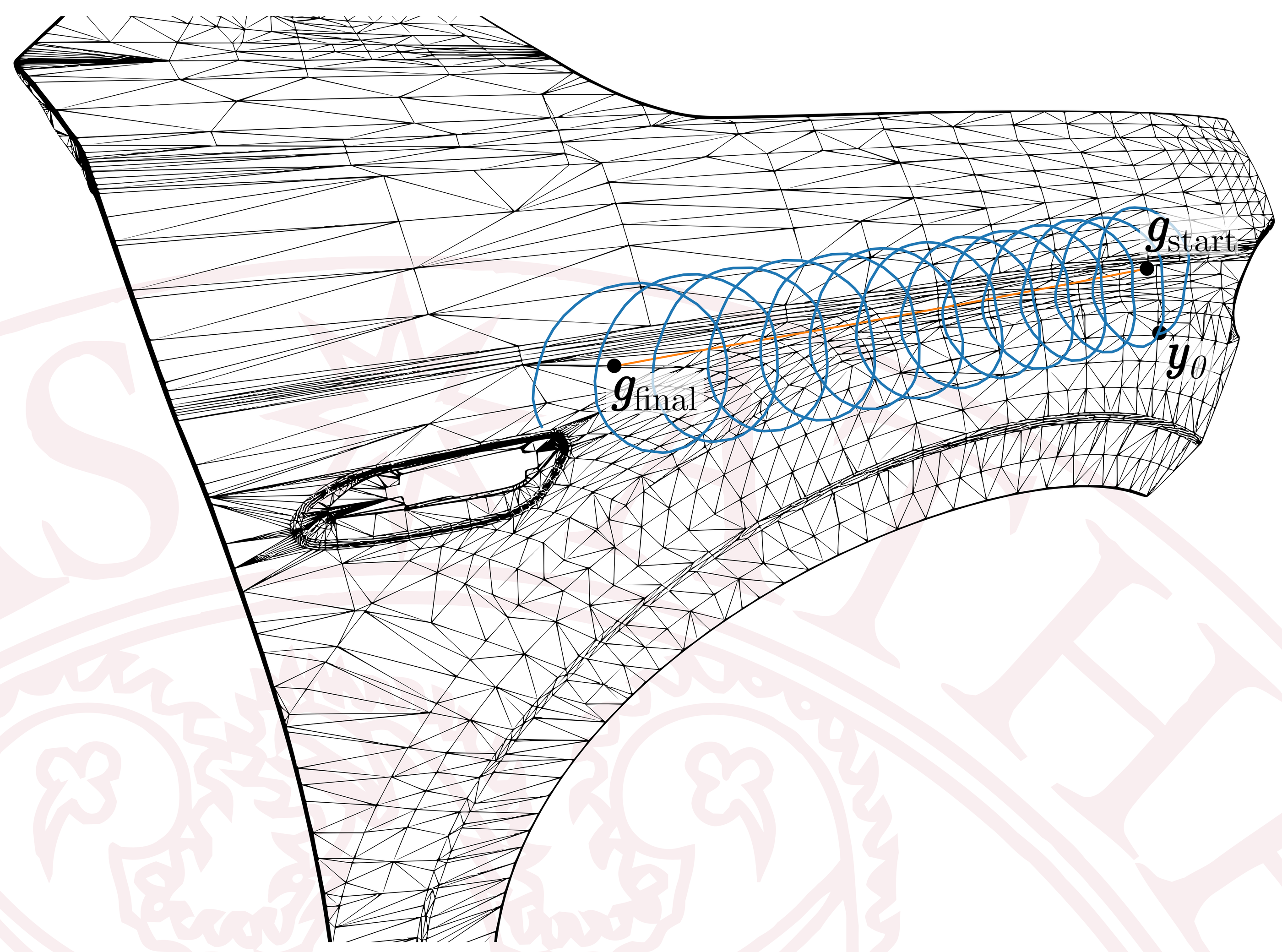
- ▷ on $\mathcal{S}$, $\boldsymbol x_2 = \operatorname{Exp}_{\boldsymbol x_1} : T_{\boldsymbol x_1}\mathcal{S} \to \mathcal{S}$ is the inverse operation of the logarithmic map. It is achieved by *wrapping* $\boldsymbol v \in T_{\boldsymbol x_1}\mathcal{S}$ along the surface $\mathcal{S}$;
- ▷ on $\mathcal{M}$, we propose an algorithm that performs the *folding* of a vector $\boldsymbol w \in T_{\boldsymbol m_1}\mathcal{M}$ to get the exponential $\boldsymbol m_2 = \operatorname{Exp}_{\boldsymbol m_1}(\boldsymbol w)$.

Results

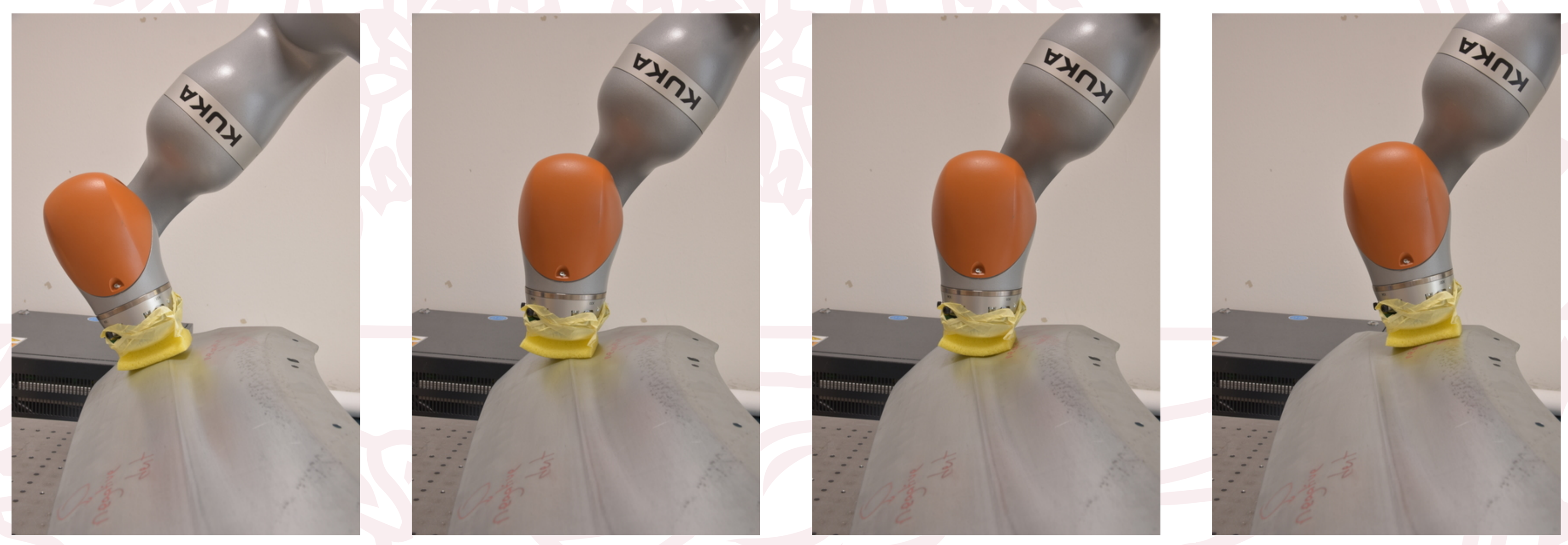
The framework can **learn** and **reproduce** policies on **arbitrary surfaces**. But it can also **generalise** to other surfaces:



What if we take a **circular motion primitive** and we **shift its center** as we move? We get a **polishing**-like behavior:



By planning directly on the mesh, the tool can **smoothly adapt** to the underlying surface:



References

[1] A.-D. Fares J., M. Saveriano, and V. Kyrki, "A unified formulation of geometry-aware discrete dynamic movement primitives," *Neurocomputing, vol. 598, 2024.*

[2] *S.-Q. Xin and G.-J. Wang, "Improving chen and han's algorithm on the discrete geodesic problem," ACM Transactions on Graphics, vol. 28, no. 4, 2009.*

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